9.1.1 The two assumptions for our Bell Inequality

We introduced the **Bell inequality** for the CHSH game, stating

$$WR_C \leq 3/4. \tag{63}$$

One quick way to prove this is to take all possible input output mappings, and count the number of strategies that allow Alice and Bob to win. A quick Python script could verify this, and you will find that the only 75% of the mappings will allow Alice and Bob to win.

But what exactly is a Bell inequality arguing, and why is violating the Bell inequality a proof of quantumness? Bell inequalities are defined based on a set of "reasonable" assumptions, of which there are two for the particular one we see here.

1. **Locality**: This assumption states that the action in one location can not instantly affect change in another location.

2. **Realism** (Classical objects): This assumption states that all objects with properties that can be measured, have those properties whether we look at them or not. Just because you leave your room, the color of your desk doesn't change! This should be true for properties that are harder to measure too...

As an expression, we have the following:

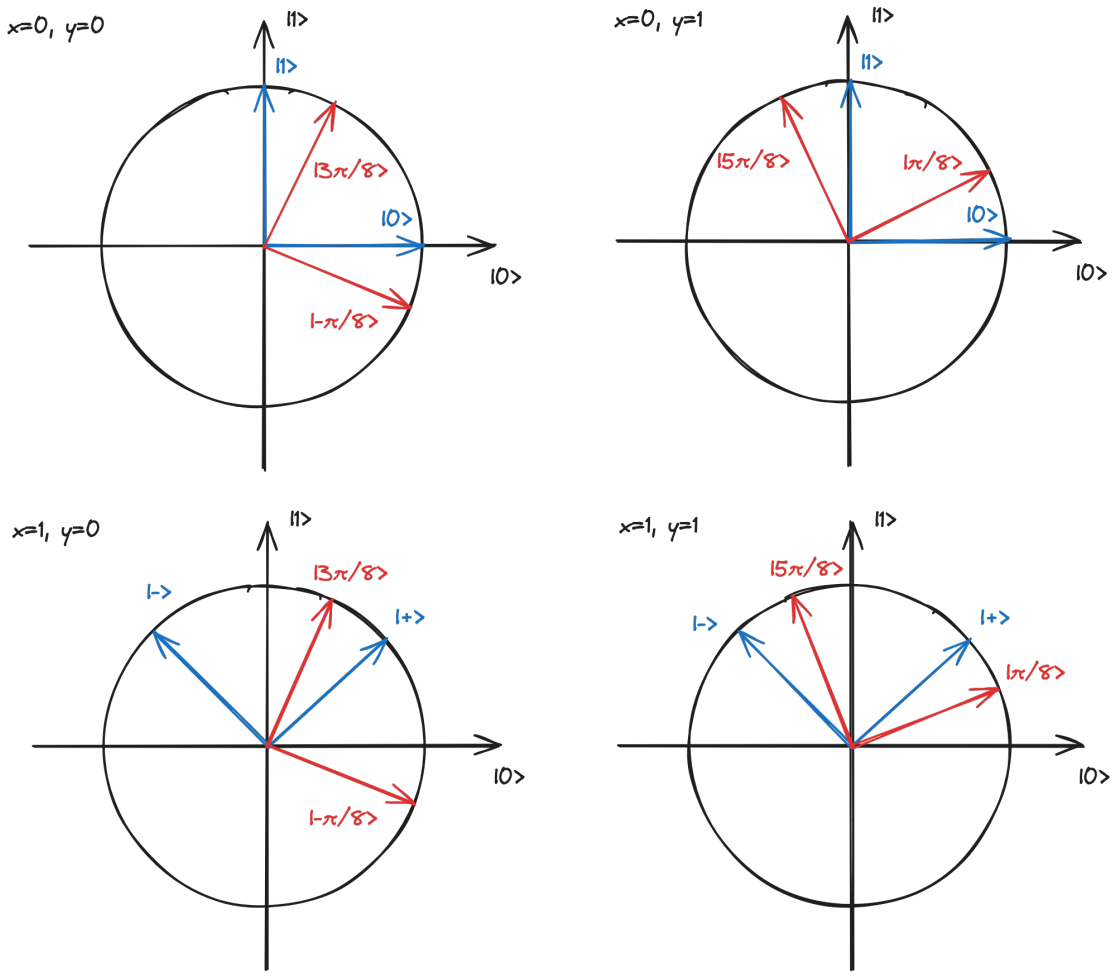
If locality and realism hold for everything in this universe, then equation (63) is true.

If (63) is not true, then one of our assumptions must be wrong...

Alice and Bob decide on the following strategy.

Alice	Bob
If $x = 0$, measure in $\{\|0\rangle, \|1\rangle\}$ basis.	If $y = 0$, measure in $\{\|-\pi/8\rangle, \|3\pi/8\rangle\}$ basis.
If $x = 1$, measure in $\{\|+\rangle, \|-\rangle\}$ basis.	If $y = 1$, measure in $\{\|\pi/8\rangle, \|5\pi/8\rangle\}$ basis.
If the outcome is $\|0\rangle$ or $\|-\rangle$, output $a = 0$.	If the outcome is $\|\pi/8\rangle$ or $\|-\pi/8\rangle$, output $b = 0$.
If the outcome is $\|1\rangle$ or $\|+\rangle$, output $a = 1$.	If the outcome is $\|3\pi/8\rangle$ or $\|5\pi/8\rangle$, output $b = 1$.

The following figure draws the bases that Alice and Bob will measure in depending on the results of the random bits.



Question 84. What is the probability that Alice and Bob win against Charlie if both of the random bits they receive are 0?

9.2 Generating Random Numbers

"Einstein, stop telling God what to do."

- Neils Bohr

Initially, the Bell inequality was taught because it was conceptually important. The CHSH game was only useful as a thought experiment, not something with real applications. However, researchers have come up with some applications for the CHSH game.

Generating *truly* random numbers is an important task in computing, especially for cryptography. Classically, we rely on pseudorandom number generators, which only mimic randomness up to some level of undetectability.

Quantum computers could be useful for generating random numbers.

Question 85. Design a single qubit circuit that outputs a random bit.

Suppose someone started selling "qubit boxes" on Amazon. The seller said you can purchase these and entangle them using their other products too to build your own quantum computer. How can you check that these are truly qubits, and not just mimicing qubit behavior?