$\alpha|0\rangle + \beta|1\rangle$

10.5 The Shor code

The two codes we saw so far are only robust to one type of error. However, we know that realistically if a system is prone to one type of error, there is nothing stopping it from being affected by another type of error too! This is where the **Shor code** comes in. This is a code that can protect againt the effects of an arbitrary error on a single qubit.

To get there, we first encode the qubit using the phase flip code:

- $|0\rangle \to |+++\rangle$
- $|1\rangle \to |---\rangle$

$\alpha|+++\rangle + \beta|---\rangle$

We then encode *each* of these qubits using the three qubit bit flip code:

- $|+\rangle \to \frac{1}{\sqrt{2}}(|000\rangle + |111\rangle)$
- $|-\rangle \to \frac{1}{\sqrt{2}}(|000\rangle - |111\rangle)$

$|+\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$

The result is a nine qubit code, with codewords given by

Z

$$|0\rangle \to |0_L\rangle \equiv \frac{(|000\rangle + |111\rangle)(|000\rangle + |111\rangle)(|000\rangle + |111\rangle)}{2\sqrt{2}} \tag{76}$$

No.

$$|1\rangle \to |1_L\rangle \equiv \frac{(|000\rangle - |111\rangle)(|000\rangle - |111\rangle)(|000\rangle - |111\rangle)}{2\sqrt{2}} \tag{77}$$

$\alpha|0\rangle + \beta|1\rangle \mapsto \alpha|0_L\rangle + \beta|1_L\rangle$

$Z_1Z_2 \quad + \quad Z_2Z_3$

$Z_4Z_5 \quad + \quad Z_5Z_6$

$Z_7Z_8 \quad + \quad Z_8Z_9$

Do blocks 1 and 2 have the same relative sign?

$X_1X_2X_3X_4X_5X_6$

$X_4X_5X_6X_7X_8X_9$

Do blocks 2 and 3 have the same relative sign?

No.

Question 97. What are the observables we should measure on the Shor code to determine if an error has occurred?